WORLD 3 / CHAPTER 2

BONDS

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BONDS

1. Terminology

Bond / debenture

A bond/debenture is a negotiable debt. The issuer obliges himself to pay the owner a specific interest rate for an agreed period of time and to repay the principal on a specified settlement day (or on several settlement days).

These debentures can be divided along many different criteria. We would like to go into some of them and their forms, first:

I. Issuer

Bonds can be categorized in terms of their issuer into

- government bonds
- bank bonds
- corporate bonds

II. Primary market

- domestic bonds: bonds that are issued by the governmentt, banks, or corporations in their own, domestic market
- foreign bonds: bonds that are issued in the domestic market by a foreign institution; e.g. a bond that is issued by a US bank in Germany. Foreign bonds carry often typical names. In the UK bulldog, in Spain Matador, in the US Yankee, in Japan Samurai and in Switzerland Alpine.
- eurobonds or international bonds: bonds that are issued in the international market under the regulations of the "International Securities Market Association".

Due to the increasing deregulation of domestic markets the differentiation between domestic, foreign and euro-bonds is becoming obsolete.

III. Registered bonds / bearer bonds

A bond might be issued as either a registered or a bearer bond.

- Registered bonds: The current owner of the bond and every change of ownership is recorded in a central register. Coupon payments and redemption are booked on the account of the current owner.

- Bearer bonds: The current bearer of the bond is entitled to receive the interest payments and redemption. The current owner is not registered. Interest payments are made to the bearer of the interest coupons and the repayment is to be made to the bearer or sender of the bond.

Interest payments

There exist fixed-rate interest bonds on the one hand and floating-rate interest bonds ("floating-rate notes", FRNs) on the other hand.

With fixed-rate interest bonds, the interest rate (in per cent of the nominal value) is fixed at the time of issue and does not change during the bond's term.

With floating-rate interest bonds, the interest rate is defined at a premium or discount to a specific reference/benchmark rate (e.g. LIBOR, EURIBOR....). Due to possible changes of the reference rate, the interest payments may vary during the bond's term. The fixing of the interest rate for the next period is done at pre-defined dates.

Another criterion to distinct bonds is the frequency of the interest payments. The following types of bonds can be found in the market:

- With zero-bonds, no interest payments take place during the bond's term. The whole interest payment (including compound interest) is done at maturity.
- With annual interest payments, a payment has to be made annually.
- With non-annual interest payments, payments take place either quarterly or six-monthly.

Redemption

Usually, the issuer of a bond obliges himself to pay off the capital at a pre-set date and at a defined rate (usually at a rate of 100). If the whole bond is paid off at once it is called a "bullet". If the bond is paid off gradually, it is called an amortizing bond. The possibility to redeem a part of the principal in advance is called ”sinking fund”.

In addition to the bonds mentioned above, there are the so-called callable bonds, where the issuer has the right to pay off the bond before maturity at a prior defined rate.

Convertible bonds

Convertible bonds are unsecured fixed-rate bonds that give the owner the right (but not the obligation) to convert the bonds into shares (under conditions that have been specified in advance). Usually, the interest rate for such convertible bonds is below the interest rate for "normal" bonds, since the owner has the right to convert the bond into shares.

Face value

Every bond has a fixed face value. This face value serves

- as the basis for the interest payment
- as the basis to calculate the redemption value which can differ from the face value. If the current price of the bond is above (below) 100, the bond is said to be quoted at a premium (discount).

Quotation

Bond prices are usually quoted in per cent of the face value.

For example, the bond's price of 101.50 for CHF government bonds corresponds to the price of CHF 101.50 per cent of the nominal value.

In the eurobonds' market, prices are usually quoted in decimals (e.g. 101.50), while in the US and in Great-Britain they are often quoted with fractions (e.g. 101½ or $101^{16}/_{32}$).

In the bond market, market makers quote both bid and offer on request: the bid price is the price they are willing to pay for bonds while the offer price is the price at which they are willing to sell the bonds.

Accrued interest

The owner of the bond receives the full amount of interest at coupon dates, even though he might not have possessed the bond during the whole period of interest. Therefore, when a bond is sold or bough, accrued interest has to be taken into account. Since comparing bond prices, that include accrued interest, is a complicated business, bonds are usually quoted without accrued interest. Nonetheless, the buyer of a bond has to pay the accrued interest to the seller when he purchases the bond

The price without accrued interest is called "clean" price.

The price including accrued interest is called "dirty" price.

To calculate the accrued interest, one computes the interest on the face value for the elapsed days(taking into account the respective method of calculation).

Example

A CAD bond with a 7 %-coupon and a time to maturity of 3½ years is sold for 101.50. Since the last coupon date, 6 months have elapsed. Therefore, the buyer of the coupon must pay

$$101.50 + 100 \cdot 0.07 \cdot \frac{180}{360} = 105.00 .$$

Pricing of fixed-rate interest bonds with given redemption schedules

The price of a fixed-rate interest bond corresponds to the price at which market participants are willing to buy or sell the bond. When speaking about the price of a bond, one usually refers to its market price.

The market price is influenced by the following factors:

1. the time to maturity of the bond
2. the actual market yield of bonds with the same time to maturity
3. the fixed interest rate of the bond
4. the linked credit risk (credit quality of the issuer)
5. the liquidity of the bond's secondary market

We will concentrate on the first three of these factors (time to maturity, yield, and coupon). We use government bonds for demonstration purposes, because here the influence of both the premium for credit risks and a possible liquidity premium are reduced to a minimum.

If interest rates change, bond prices change

Assume that a bond has been issued at 100 and at a fixed rate of 7 %. Since the coupon that has been fixed at the issue date does not change, the bond (with an unchanged price) looses its appeal to investors if the interest rate in the market rises. With a market yield of, e.g., 8 % few investors can be found who are interested in the bond if the current price still remains at 100. For this reason, the price of the bond falls if the interest rate rises in order to offer the investor an equally attractive investment opportunity.

On the other hand, a decrease in interest rates would lead to a rise in the bond price. In the example above, the 7 %-bond with an unchanged price of 100 and a current market yield would offer an extremely attractive investment opportunity for any investor. The increased demand will lead to rising bond prices.

Interest rates	↗	Price	↘
Interest rates	↘	Price	↗

The current, fair price of a bond is the sum of the discounted cash flows that result from this bond.

Fixed-rate interest "bullet" bonds with annual coupons

The common formula used in the pricing of a bullet bond is as follows:

$$P = \left\{ \left(C \cdot \sum_{n=1}^{N} \frac{1}{(1+r)^{n}} \right) + \frac{1}{(1+r)^{N}} \right\} \cdot 100$$

P = bond price
C = coupon, in decimals (6% = 0.06)
r = current market yield, in decimals
n = ongoing year
N = total number of years

Example

Calculate the price for a CHF government bond with a 7 %-coupon, 5 years to maturity, and annual interest payments (the latest payment just took place). The current interest rate (yield) for government bonds with 5 years to maturity is 6 %.

$$P = \left(0.07 \cdot \sum_{n=1}^{5} \frac{1}{(1+0.06)^{n}} + \frac{1}{(1+0.06)^{5}} \right) \cdot 100$$

$$= (0.07 \cdot 4.21237 + 0.74726) \cdot 100$$

= 104.21

We demonstrate the calculation of the sum in the following table:

Year	$\frac{1}{(1+0.06)^n}$
1	$\frac{1}{(1+0.06)^1}=0.94340$
2	$\frac{1}{(1+0.06)^2}=0.89000$
3	$\frac{1}{(1+0.06)^3}=0.83962$
4	$\frac{1}{(1+0.06)^4}=0.79209$
5	$\frac{1}{(1+0.06)^5}=0.74726$
Sum:	4.21237

2. General formula for pricing

Since the amount of capital is not necessarily paid back at maturity and the interest payments are not necessarily paid annually, a number of specialized formulae for pricing exists. For this reason, we present a general formula that takes into account different types of interest payments as well as different types of repayment arrangements.

First, we determine the cash flows that result from a nominal amount of 100. Then, these cash flows are discounted by the current yield.

$$\sum_{n=1}^{N} \frac{1}{(1+r)^{n}} \cdot CF_{n}$$

r = current yield, in decimals

n = ongoing period

N = total number of periods

CF_n = cash flow (at a nominal of 100) at time n

Since pricing does not necessarily happen at coupon dates (result is a "broken" period at the beginning), therefore we have to generalize the formula again:

$$\left[\frac{CF_1}{1+r \cdot \frac{d}{B}} + \sum_{n=2}^{N} \frac{CF_n}{\left(1+\frac{r \cdot d}{B}\right) \cdot (1+r)^{n-1}} \right] - AI$$

d = days to the first cash flow

AI = $\text{accrued interest} \cdot \frac{\text{coupon days} - d}{\text{coupon days}}$

B = Day base (360/ 365/ ACT)

Note: With "broken" periods there are different ways of calculation. In Germany, the "Moosmüller-Methode" is usually employed, i.e. the first discounting is calculated linearly (1+r · d/B); internationally, the "ISMA-method" is commonly used, which discounts cash flows exponentially, i.e. (1+r ·d/B) is replaced with $(1+r)^{d/B}$.

2.1. Examples

Example

Price of a EURO government bond with the following specifications

coupon:	7.00 % fixed
time to maturity:	5 years
interest payment:	annual
redemption:	bullet
current yield:	6.00 %

Assumption: The latest interest payment has just been made.

1 Year	2 Cash flow	3 Discounting	4 Present value (2·3)
1	7	$\frac{1}{(1+0.06)^1}=0.94340$	6.6038
2	7	$\frac{1}{(1+0.06)^2}=0.89000$	6.2300
3	7	$\frac{1}{(1+0.06)^3}=0.83962$	5.8773
4	7	$\frac{1}{(1+0.06)^4}=0.79209$	5.5447
5	107	$\frac{1}{(1+0.06)^5}=0.74726$	79.9566
		Sum:	104.2124

Example

Price of a EURO government bond with the following specifications

coupon:	7.00 % fixed
time to maturity	4 years**, 270 days**
interest payment:	annual
redemption:	bullet
current yield:	6.00 %

1 *Year*	*2* *Cash flow*	*3* *Discounting*	*4* *Present value (2·3)*
270 days	7	$\frac{1}{\left(1+0.06\cdot\frac{270}{365}\right)}=0.95750$	6.7025
1 year, 270 days	7	$\frac{1}{\left(1+0.06\cdot\frac{270}{365}\right)\cdot(1+0.06)^1}=0.9033$	6.3231
2 years, 270 days	7	$\frac{1}{\left(1+0.06\cdot\frac{270}{365}\right)\cdot(1+0.06)^2}=0.8522$	5.9652
3 years, 270 days	7	$\frac{1}{\left(1+0.06\cdot\frac{270}{365}\right)\cdot(1+0.06)^3}=0.8039$	5.6276
4 years, 270 days	107	$\frac{1}{\left(1+0.06\cdot\frac{270}{365}\right)\cdot(1+0.06)^4}=0.7584$	81.1522
Sum (dirty price):			105.7706
Accrued interest: $7\cdot\frac{95}{365}$			1.8219
Price (clean price):			103.9487

Note: with the ISMA-Method the price is 103,9830.

Example

Price of a EUR government bond with the following specifications

coupon:	7.00 % fixed
time to maturity:	5 years
interest payment:	**semi-annually**
redemption:	bullet
current yield:	6.00 % (=3.0 % s.a.)

Assumption: The latest interest payment just took place.

1 Year	2 Cash-flow	3 Discounting	4 PV (2·3)
0.5	3.5	$\frac{1}{(1+0.03)^1}=0.9708738$	3.3980583
1	3.5	$\frac{1}{(1+0.03)^2}=0.9425959$	3.2990857
1.5	3.5	$\frac{1}{(1+0.03)^3}=0.9151417$	3.2029958
2	3.5	$\frac{1}{(1+0.03)^4}=0.8884871$	3.1097047
2.5	3.5	$\frac{1}{(1+0.03)^5}=0.8626088$	3.0191308
3	3.5	$\frac{1}{(1+0.03)^6}=0.8374843$	2.9311949
3.5	3.5	$\frac{1}{(1+0.03)^7}=0.8130915$	2.8458203
4	3.5	$\frac{1}{(1+0.03)^8}=0.7894092$	2.7629323
4.5	3.5	$\frac{1}{(1+0.03)^9}=0.7664167$	2.6824586
5	103.5	$\frac{1}{(1+0.03)^{10}}=0.7440939$	77.01372
Sum:			104.2651

Example

Price of a EUR government bond with the following specifications

coupon:	7.00 % fixed
time to maturity:	5 years
interest payment:	annual
redemption:	**50 % after 3 years, rest: at end of term**
current yield:	6.00 %

Assumption: The latest interest payment just took place.

1 ***Year***	***2*** ***Cash flow***	***3*** ***Discounting***	***4*** ***Present value (2·3)***
1	7	$\frac{1}{(1+0.06)^1} = 0.94340$	6.6038
2	7	$\frac{1}{(1+0.06)^2} = 0.89000$	6.2300
3	57	$\frac{1}{(1+0.06)^3} = 0.83962$	47.8583
4	3.5	$\frac{1}{(1+0.06)^4} = 0.79209$	2.7723
5	53.5	$\frac{1}{(1+0.06)^5} = 0.74726$	39.9783
		Sum:	103.4427

Note: All of the above calculations are based on the assumption of a flat yield curve, i.e. same interest rates for all terms.

3. Pricing of fixed-rate interest bonds with the zero-curve

In the above calculations we assumed that coupons, paid during the term of the bond could be re-invested at the same rate. With a flat yield curve, this is true.

Theoretically, one should use a calculation based on the so-called zero-curve that is already in use in the swap market, but has not yet been accepted in the bond market.

With the zero bond method, every payment is regarded separately. This implies that there are no interest payments during the time to the settlement date. When calculating the bond price with the zero-curve, each cash flow is treated like a cash flow from a zero bond. This way, different interest rates for different terms are taken into account during the price calculation.

Formula (simplified for "clean" terms)

$$\sum_{n=1}^{N} \frac{1}{(1+Z_n)^n} \cdot CF_n$$

Z_n = zero rate at time n
n = ongoing year
N = total number of years
CF_n = cash flow at time n

Excursus: Pricing of zero-rates

Example

As in the previous example, we assume the following yields for the bonds:

year	interest rate
1	4.00 %
2	4.50 %
3	5.00 %

With a term of 1 year, there is no interest payment during the term (annual payments assumed). Therefore, by definition the one-year zero rate (Z_1) is the same as the one-year yield; i.e. here 4.00 %.

To calculate the two-year zero rate, one splits a two-year bond into two zero transactions.

Cash flows at a purchase of a two-year bond:

SPOT

+	-
	100

1 yr

+	-
4.50	

2 yrs

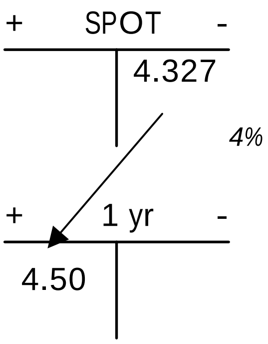
+	-
104.50	

a) Computing the cash flow after 1 year by a one-year zero (present value calculation).

$$\text{initial capital} = \frac{\text{final capital}}{(1 + Z_1)}$$

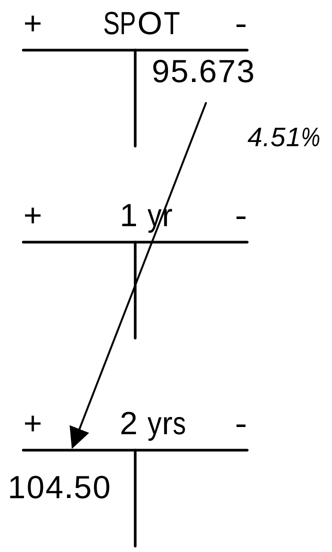
$$4.327 = \frac{4.50}{1.04}$$

To earn an amount of EUR 4.50 after 1 year, one has to invest EUR 4.327 in a 1-year zero.



b) Computing the remaining cash flows by a 2-year zero (zero yield calculation):

Since EUR 4.327 were necessary to receive the cash flow after one year, the initial capital for the two-year cash flow is reduced to EUR 95.673 (= 100 - 4.327).



That means:

$$\text{initial capital} \cdot (1 + Z_2)^2 = \text{final capital}$$

$$95.673 \cdot (1 + Z_2)^2 = 104.5$$

Therefore:

$$Z_2 = \left(\left(\frac{104.5}{95.673} \right)^{\frac{1}{2}} - 1 \right)$$

$$Z_2 = 4.51\%$$

With the given yields, the 2-year zero will be at 4.51 %.

You use the same method (with the respective values) to calculate the 3-year zero bond:

One splits the 3-year bond (yield: 5.00 %) into three zero-transactions. By transferring the 1-year and 2-year cash flows into 1-year and 2-year zeros you get the amount of the initial capital that has to be earned by the cash flow after three years. With these two inputs you can calculate the 3-year zero (i.e. here: 5.034 %).

Knowing the 3-year zero , you can calculate the 4-year zero and so on....

General formula to calculate with zero rates:

$$Z_N = \left(\frac{1 + r_N}{1 - \sum_{n=1}^{N-1} \frac{r_N}{1 + Z_n}} \right)^{\frac{1}{N}} - 1$$

r_N = bond yield for a term of N years, in decimals
Z_n = zero rate for the term of n years, in decimals
N = total number of years
n = ongoing year

Note: Only recursive calculation is possible: to get the 5-year zero-rate you first have to calculate the zeros for the years 1 through 4.

Example

Price of a EUR government bond with the following specifications

coupon:	7.00 % fixed
time to maturity:	5 years
interest payment:	annual
redemption:	bullet
current yield:	6.00 %

Assumption: The latest interest payment has just been made.

Zero of the example

Example

Year	*Rate of interest*	*Zero rate*
1	4.00 %	4.00 %
2	4.50 %	4.51 %
3	5.00 %	5.03 %
4	5.50 %	5.57 %
5	6.00 %	6.13 %

1 Year	*2 Cash flow*	*3 Zero rate*	*4 Discounting*	*5 Present value (2·3)*
1	7	4.00 %	$\frac{1}{(1+0.04)^1}=0.96154$	6.7308
2	7	4.51 %	$\frac{1}{(1+0.0451)^2}=0.91555$	6.4089
3	7	5.03 %	$\frac{1}{(1+0.0503)^3}=0.86310$	6.0417
4	7	5.57 %	$\frac{1}{(1+0.0557)^4}=0.8050$	5.6355
5	107	6.13 %	$\frac{1}{(1+0.0613)^5}=0.74269$	79.4681
			Sum:	104.2850

Taking into account the yield curve for the price calculation, you get a theoretical price difference this example of approx. 7 basis points (104.2124 to 104.2850).

4. Calculation of price sensitivities/ the concept of duration

In the previous section we looked at the bond pricing formulae and developed an attractive instrument to evaluate different bonds. Next, we want to be able to estimate the price sensitivity of bonds.
For dealers of short-term positions in bonds (trading book) it is useful to estimate how the bond' price changes when the underlying factors change.
According to the previous section, the main factors are:

I. the time to the bond's maturity
II. the actual market yield of bonds with the same time to maturity
III. the fixed interest rate of the bond
IV. the linked credit risk
V. the liquidity of the market.

Again, we use government bonds for our analysis and thus can ignore factors 4 and 5 (credit risk and liquidity).

The obvious question is how the bond price changes when the underlying interest rates change. Since we are interested in how the price of a particular bond changes, coupon payments as well as the bond's term remain constant.

The best known method to determine the price sensitivity is the concept of duration (or rather, the so-called "modified duration").

The modified duration shows how bond prices are affected by a change in the market interest rates.

If a bond (with a current dirty price of 104.00) has a modified duration of, e.g., 3.50, it means that a change in the interest rates by 1 % brings about a change in the bond price by

$$\frac{104 \cdot 3.50\%}{100} = 3.64\%.$$

In combination with our rules concerning price changes in the wake of interest rate changes, we receive the following result:

- if interest rates rise by 1 %, the price of the bond falls by 3.64 %
- if interest rates falls by 1 %, the price of the bond rises by 3.64 %.

$$CP = (-MD) \cdot P \cdot CY$$

CP = change of the bond price
MD = modified duration
P = price of the bond (dirty price)
CY = change of the market yield in decimals (1% = 0.01)

Example

Specifications of a fixed-rate interest bond

current price	104.00
modified duration	3.50
current yield	6.00 %

What will be the bond price at a market yield of 5.75 % or 6.75 %?

- at 5.75 %:

$$CP = (-3.50) \cdot 104.00 \cdot (-0.0025) = +0.91$$

At a market rate of 5.75 % the price of the bond will be 104.91 (104.00 + 0.91).

- at 6.75 %:

$$CP = (-3.50) \cdot 104.00 \cdot (+0.0075) \quad = -2.73$$

At a market rate of 6.75 % the price of the bond will be 101.27 (104.00 – 2.73).

Computing the modified duration

$$MD = \frac{\sum_{n=1}^{N} n \cdot CF_n \cdot \frac{1}{(1+r)^n}}{\sum_{n=1}^{N} CF_n \cdot \frac{1}{(1+r)^n}} \cdot \frac{1}{(1+r)}$$

MD = modified duration

n =ongoing year

N = total number of years

CF_n = cash flow (at a nominal of 100) at time n

r = current yield in decimals

4.1. Examples

Example

Bond with the following specifications

time to maturity	5 years
coupon:	7.00 %
current yield:	6.00 %
interest payment:	annual
redemption:	bullet

1 *Year*	*2* *Cash flow*	*3* *Discounting*	*4* *Present value of weighted cash flow (1·2·3)*	*5* *Present value of cash flow (2·3)*
1	7	$\frac{1}{(1+0.06)^1}=0.94340$	6.6038	6.6038
2	7	$\frac{1}{(1+0.06)^2}=0.89000$	12.4600	6.2300
3	7	$\frac{1}{(1+0.06)^3}=0.83962$	17.6320	5.8773
4	7	$\frac{1}{(1+0.06)^4}=0.79209$	22.1786	5.5447
5	107	$\frac{1}{(1+0.06)^5}=0.74726$	399.7831	79.9566
		Sum:	458.6575	104.2124

$$MD=\frac{458.6575}{104.2124}\cdot\frac{1}{(1+0.06)} = 4.15$$

Example

Bond with the following specifications

time to maturity:	5 years
coupon:	7.00 %
current yield:	6.00 %
interest payment:	annual
redemption:	**50 % after 3 years, rest: end of term**

1 *Year*	*2* *Cash flow*	*3* *Discounting*	*4* *Present value of weighted cash flow (1·2·3)*	*5* *Present value of cash flow (2·3)*
1	7	$\frac{1}{(1+0.06)^1}=0.94340$	6.6038	6.6038
2	7	$\frac{1}{(1+0.06)^2}=0.89000$	12.4600	6.2300
3	57	$\frac{1}{(1+0.06)^3}=0.83962$	143.5749	47.8583
4	3.5	$\frac{1}{(1+0.06)^4}=0.79209$	11.0893	2.7723
5	53.5	$\frac{1}{(1+0.06)^5}=0.74726$	199.8916	39.9783
		Sum:	373.6195	103.4427

$$MD=\frac{373.6195}{103.4427}\cdot\frac{1}{(1+0.06)} = 3.41$$

Example

Bond with the following specifications

time to maturity:	5 years
coupon:	7.00 %
current yield:	6.00 %
interest payment:	**semi-annually**
redemption:	bullet

1 *Year*	*2* *Cash-flow*	*3* *Discounting*	*4* *PV (2·3)*	*5* *weighted PV*
0.5	3.5	$\frac{1}{(1+0.03)^1}=0.9708738$	3.3980583	1.6990
1	3.5	$\frac{1}{(1+0.03)^2}=0.9425959$	3.2990857	3.2991
1.5	3.5	$\frac{1}{(1+0.03)^3}=0.9151417$	3.2029958	4.8045
2	3.5	$\frac{1}{(1+0.03)^4}=0.8884871$	3.1097047	6.0383
2.5	3.5	$\frac{1}{(1+0.03)^5}=0.8626088$	3.0191308	7.5478
3	3.5	$\frac{1}{(1+0.03)^6}=0.8374843$	2.9311949	8.7936
3.5	3.5	$\frac{1}{(1+0.03)^7}=0.8130915$	2.8458203	9.9604
4	3.5	$\frac{1}{(1+0.03)^8}=0.7894092$	2.7629323	11.0475
4.5	3.5	$\frac{1}{(1+0.03)^9}=0.7664167$	2.6824586	12.0711
5	103.5	$\frac{1}{(1+0.03)^{10}}=0.7440939$	77.01372	385.0686
Sum:			104.2651	450.3298

$$MD=\frac{450.3298}{104.2651}\cdot\frac{1}{(1+0.06)}=4.07$$

To sum up, we can state the following rules regarding price sensitivity of fixed-rate interest bonds:

High duration at (high price sensitivity)	→	long term
	→	low coupon
Low duration at (low price sensitivity)	→	short term
	→	high coupon

Note 1: While calculating the duration, we assumed a flat yield curve (just as we did when calculating prices). A calculation of the price sensitivity may also take into account the effects of the yield curve: in this case, we speak of "effective duration". Nevertheless, the yield curve's influence on the duration is relatively small, so we are able to do without calculating the effective duration here.

Note 2: The modified duration also implies that price changes are linear, i.e. the duration remains the same while the interest level changes. This leads us to the concept of convexity that demonstrates how the duration changes if the interest rates change (similar to the gamma-factor with options). However, the magnitude of the price changes can be estimated quite well by the modified duration, so we are also able to do without convexity and its mathematical explanation here.