WORLD 3 / CHAPTER 1

FINANCIAL MATHEMATICS

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1. Converting from money market basis to bond basis and vice versa

Since the basis for interest payments usually is different in the capital market and the money market, we must be able to convert these payments.

Assume that we know the money market's interest rates. If we want to calculate the interest rate in the capital market, we use the following formula:

$$r_{CM} = r_{MM} \cdot \frac{D_{MM}}{B_{MM}} \cdot \frac{B_{CM}}{D_{CM}}$$

r_{CM} = interest rate in the capital market

r_{MM} = interest rate in the money market

D_{MM} = number of days per year, money market

B_{MM} = basis of term calculation, money market

D_{CM} = number of days per year, capital market

B_{CM} = basis of term calculation, capital market

The following formula is used to convert an interest rate of the capital market into one of the money market:

$$r_{MM} = r_{CM} \cdot \frac{D_{CM}}{B_{CM}} \cdot \frac{B_{MM}}{D_{MM}}$$

Example

The interest rate in the capital market (basis 30 / 360) is to be compared to an interest rate in the money market.

CHF interest rate capital market: 3.50 %

17 April 1997 - 17 April 1998 (365 days) =

$$3.50\% \cdot \frac{360}{360} \cdot \frac{360}{365} =$$

3.45205% money market

25 April 1997 - 27 April 1998 (367 days) =

$$3.50\% \cdot \frac{362}{360} \cdot \frac{360}{367} =$$

3.45232% money market

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2. Calculating the effective interest rate (non-annual payments)

Interest payments are not always due annually but daily, weekly, monthly, quarterly, and semi-annual interest payments are also possible. With bonds, it is quite common that interest payments are made semi-annually. To be able to compare these non-annual interest payments to yearly payments (single payment of interest), one converts the nominal interest rate into the effective interest rate. With a single p.a. payment, the nominal rate equals to the effective rate of interest.

$$ER = \left(1 + \frac{NR}{FIP}\right)^{FIP} - 1$$

ER = effective rate of interest p.a. in decimals
NR = nominal rate of interest p.a. in decimals
FIP = frequency of interest payments p.a. (e.g. 2 for semi-annual, 4 for quarterly,....etc.)

Example

Calculate the effective p.a. interest rate from the nominal p.a. interest rate, that is based on quarterly interest payments.

$$ER = \left(1 + \frac{0.06}{4}\right)^{4} - 1 = 6.13636$$

Therefore, a quarterly interest rate of 6% is equivalent to an annual interest rate of 6.14 %.

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3. Conversion of annual interest payments into non-annual interest payments

We can also convert non-annual payments into annual payments.

$$r_{NA} = \left[\sqrt[FIP]{(1 + r_A)} - 1\right] \cdot FIP$$

r_{NA} = non-annual rate of interest p.a., for the term of interest
FIP = frequency of interest payments p.a.
r_A = annual rate of interest p.a., in decimals

Example

You possess a bond with annual interest payments at 6.5 %.
Calculate the nominal interest rate, based on semi-annual interest payments.

$$r_{NA} = \left[\sqrt[2]{(1 + 0.065)} - 1\right] \cdot 2 = \mathbf{6.39767}$$

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4. Calculating forward rates (non-annual)

Given a short-term and a long-term interest rate it is possible to calculate a so-called forward rate (also called forward-forward rate).

Formula:

$$FR = \left[\frac{(1+r_l)^N}{(1+r_s)^n} \right]^{\frac{1}{(N-n)}} - 1$$

FR = forward rate of interest

r_l = rate of interest in decimals, long-term

N = term in years, long term

r_k = rate of interest in decimals, short-term

n = term in years, short term

Note: The exact calculation is done on the basis of zero rates. For long terms, differences may become too big without using zeros.

Example

We use the above example, and we add the following interest structure:

2 years interest rate	4 %
3 years interest rate	4¼ %

A needed 3-year loan could not be re-financed at matching maturities, but in the first step only for 2 years. By calculating compound interest, the break-even point of the term that is not hedged can be determined.

$$FR = \left[\frac{(1+0.0425)^3}{(1+0.04)^2} \right]^{(3-2)} - 1 \quad = \mathbf{4.75180\ \%}$$

Therefore, the interest rate for the remaining unsecured term (one year, starting in 2 years) should not be higher than 4.75 %; else, the financing would have been better by taking a 3-years loan at one time.

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5. Calculating the future value

Starting at the present value (principal) the future value can be determined. Assume that an investor buys a bond with a fixed coupon and holds this bond until its maturity. The interest payments he is receiving on re-investing his coupons, are the reason that at the end of the term the total amount will be higher than the sum of the coupon payments.

Example

The **formula** for calculating the future value is:

$$PV = C \cdot (1 + r)^{N}$$

FV = future value
C = capital amount
r = interest rate p.a. in decimals
N = total term in years

Note: It is also assumed that the re-investment is done at the same rate as the coupon.

Example

The future value of a 5-years bond (coupon 6 %. price 100) is:

$$FV = 100 \cdot (1 + 0.06)^{5} = \mathbf{133.82}$$

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6. Calculating the present value

The concept of the present value is essential in the capital market. Today's value of a future cash flow is called the present value. Starting at the future value, that is known, you arrive at the present value of, for example, a bond by discounting.

As outlined in the chapter on money markets, the present value of a cash-flow in one year is calculated with the following **formula:**

$$PV = \frac{EV}{1+r}$$

FV = present value

EW = future value

r = interest rate p.a. in decimals

Example

The present value of EUR 1 in one year at a rate of 3 % is

$$PV = \frac{1}{103} = \mathbf{0.97087}$$

With a term of more than a year, the present value can be calculated in the following way:

$$PV = \frac{FV}{(1+r)^N}$$

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The present value of EUR 1 in five years at an interest rate of 6 % is:

$$PV = \frac{1}{(106)^5} = \mathbf{0.74726}$$

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7. Calculating interest from present value and future value

If present value, future value and the term of a deal are known, the interest rate can be calculated. Thereby, a re-investment of the coupon payments at the same interest rate is assumed.

$$r = \sqrt[N]{\frac{FV}{PV}} - 1$$

or

$$\left(\frac{FV}{PV}\right)^{\left(\frac{1}{N}\right)} - 1$$

FV = future value
PV = present value
N = term

Example

At what interest rate do you have to invest EUR 50 mio in order to receive EUR 100 mio (incl. accrued interest) after 10 years.

$$r = \sqrt[10]{\frac{100}{50}} - 1$$

r = 7.17735 %

Note: The re-investment of the coupon payments at the same interest rate is assumed.

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