

**FORWARD RATE AGREEMENT (FRA)**

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## FORWARD RATE AGREEMENT

The forward, or future rate agreement, is a contract between two counterparties to fix a future interest rate. This contract defines the interest rate for a future period based on a principal.

If on the agreed date (fixing date) the FRA-contract-rate differs from the agreed reference rate, a settlement payment depending on the difference must be paid by one of the contractors. The principal is not exchanged and there is no obligation by either party to borrow or lend capital. 0

### **The FRA can be used**

- by market participants who wish to hedge against future interest rate risks by setting the future interest rate today (at trading date)
- by market participants who want to make profits based on their expectations of the future development of interest rates
- by market participants who try to take advantage of the different prices of FRAs and other financial instruments, e.g. futures, by means of arbitrage.

FRAs are over the counter (OTC) products and are available for a variety of periods: starting from a few days to terms of several years. In practice, however, the FRA-market for 1-year FRAs offers the highest liquidity and is therefore also regarded as a money-market instrument.

The FRA is not an obligation to borrow or lend any capital in the future. At settlement date, the principal just serves as the basis to calculate the difference between the two interest rates, or rather the settlement payment that results from this difference.

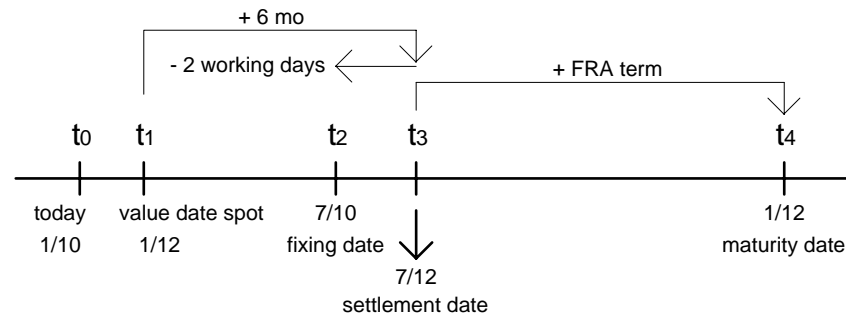
## 1. Interest Rate Derivatives

### Terminology

**Example**

spot FRA, sell 6 / 12

100 Mio 4½ %



- $t_0$ :: trading date
- $t_1$ :: value date spot
- $t_2$  : fixing date: the difference between contract rate and reference rate is determined 2 working days before settlement date.
- $t_3$  : settlement date: the settlement payment is exchanged ("amount due")
- $t_4$  : maturity date (final maturity): defines the end of the FRA period, there are no more payments to be made, but the exact term of the FRA is determined; final maturity – settlement date = days of FRA term

### FRA interest rate

The FRA interest rate is the interest rate stipulated in the contract, e.g. here 4½%.

### Reference rate

The reference rate is defined by the reference banks on the fixing date, and is usually based on LIBOR.

Though most agreements regarding documentation are settled on FRABBA-terms (of the British Bankers Association), there are also agreements on ISDA-terms (of the International Swaps and Derivatives Association).

GBP FRAs are settled on the same-day (not at value date after 2 days) and constitute therefore an exception.



**FRA purchase**

An FRA purchase is a hedge for a future short position in deposits and thus a protection against rising interest rates. Also speculation on rising interest rates in the future.

**FRA sale**

An FRA sale is a hedge for a future long position in deposits and thus a protection against falling interest rates. Also speculation on falling interest rates in the future.

**Amount due**

The amount due is the only cash-flow that exists in an FRA. It is due on settlement date. The amount due is determined by the difference between FRA rate and reference rate multiplied by the amount of capital times the FRA term and is discounted to the settlement date (because the calculation would be due on maturity date).

**Quotation**

FRA terms are usually labeled by a slash (3/9) or by a dot (3·6). The FRA terms most commonly used are terms of 3, 6, 9, or 12 months.

1/4	1/7	2/5	2/8	3/6	3/9	3/12
6/12	6/18	9/12	9/15	9/18	12/18	12/24

In contrast to interest futures, FRAs prove to be a flexible instrument. It is possible to trade "broken dates", e.g. 6-12 trade date 11th or 29th, as well as non-standard terms. A common form of "broken dates" FRAs are the so-called "IMM-dates". They refer to FRA terms that match exactly the terms of futures. Here, the largest volumes are traded.

The following terms serve as examples for non-standard FRAs:

1/2	1/3	1/5	1/6	etc.
2/3	2/4	2/6	2/7	etc.

(These are FRAs with terms of 1, 2, 4 months, etc.)

FRA-markets in USD, EUR and GBP have the most liquid one.

#### As a rule:

FRA with IR futures-underlying

→ low spreads (e.g. USD 6/12, Quotation 3.42 - 3.46)

FRA without IR futures-underlying

→ greater spreads (e.g. CZK 6/12, quotation 11.05 - 11.15)

#### Credit risk of an FRA

The FRA comprises no exchange of principal. Therefore, the only risk is the non-fulfillment of the amount due at settlement date. The credit risk is thereby limited to the difference between FRA rate and the locked-in reference rate at settlement date.

The credit line used by an FRA is usually between 2-5 % p.a. (with regard to the term of the FRA-period), from spot until settlement date. Perhaps one should also take into account the distinction between more or less volatile currencies, and the fact that currencies have different interest levels.

Purchase or sale of a 6-12 EUR 100,000,000, FRA term 182 days, FRA rate 3.45 %, limit 4 % p.a.

#### Example

$$\text{limit used} = 100,000,000 \cdot 0.04 \cdot \frac{182}{360} = \text{EUR}2,022,222.22$$

*Note:* There are also limit-systems in use that guarantee a permanent mark-to-market-valuation.

## 2. Forward Rate Agreements (FRA's)

### 2.1 The functioning of FRAs

#### FRAs work as follows:

- A customer enters into an agreement with his bank to either buy or sell an FRA. The FRA defines an interest rate for a principal of a deposit or a loan for a defined interest period that will start at a future date. The interest rate on which they agree - also known as FRA rate - is the price of the FRA as it is quoted by the market.
- By doing so, the bank has not committed itself to lend or take money at this rate. Instead, the customer and the bank agree to compare the fixed FRA rate to a reference interest rate (e.g. LIBOR) two days (exception: GBP) before the defined interest period (fixing date). The reference rate is defined on fixing date; it is also called settlement rate.
- Who receives or pays the amount due depends on whether the customer or the bank bought or sold the FRA, and whether the FRA rate is higher or lower than the reference rate at settlement date.

#### Example

A large company wishes to fix the interest rate for a loan of USD 20 Mio for 3 months, beginning in 2 months. The company might buy an FRA from a bank that is trading such instruments. The bank quotes an FRA rate. This FRA rate is applied to the principal (USD 20 Mio), but not to the 3-month loan itself. Thereby, the FRA rate serves as the fixed rate the company wanted to secure for the 3-month term of interest (from the end of the 2nd until the end of the 5th month). This fixed rate is known to both of the counterparties on trading day, but they do not know the future level of the reference rate.

Usually two days before the settlement date, the FRA rate is compared to the agreed reference rate (LIBOR).

If the reference rate is higher than the defined FRA rate, the amount due is paid to the customer. This is a compensation for the higher interest payments for his (more expensive) re-financing.

If the reference rate happens to be lower than the FRA rate, the customer must settle the balance. This effect in turn is balanced by lower interest expenses.

In this process, there is no exchange of principal; only the interest rate gaps are balanced. With the settlement payment, the interest rate for the future re-financing has been fixed at the FRA-rate.

## 2.2 The amount due

If on fixing date the FRA rate is higher than the reference rate, the FRA buyer pays to the seller. If the FRA rate is lower than the reference rate, the FRA seller pays the buyer.

In principle, the difference between the FRA rate and the reference rate is determined on the basis of the underlying principal on settlement date. The settlement date of an FRA is the first day of the defined interest period. As the FRAs are settled at the beginning of the term, while the accrued interest payments are due only at the end, the settlement payment must be discounted over the interest period.

The formula to calculate the settlement payment is as follows:

$$AD = \frac{(REF - FRA) \cdot (\pm VOL) \cdot \frac{D}{B}}{1 + \left( REF \cdot \frac{D}{B} \right)}$$

- AD = amount due
- REF = reference rate (e.g. LIBOR) in decimals
- VOL = volume of the FRA (+ = buy; - = sell)
- FRA = FRA rate in decimals
- D = number of days of the FRA term
- B = day basis of calculation

 **Example**

6/12 FRA purchase

Volume: USD 100 Mio

FRA rate: 4 %

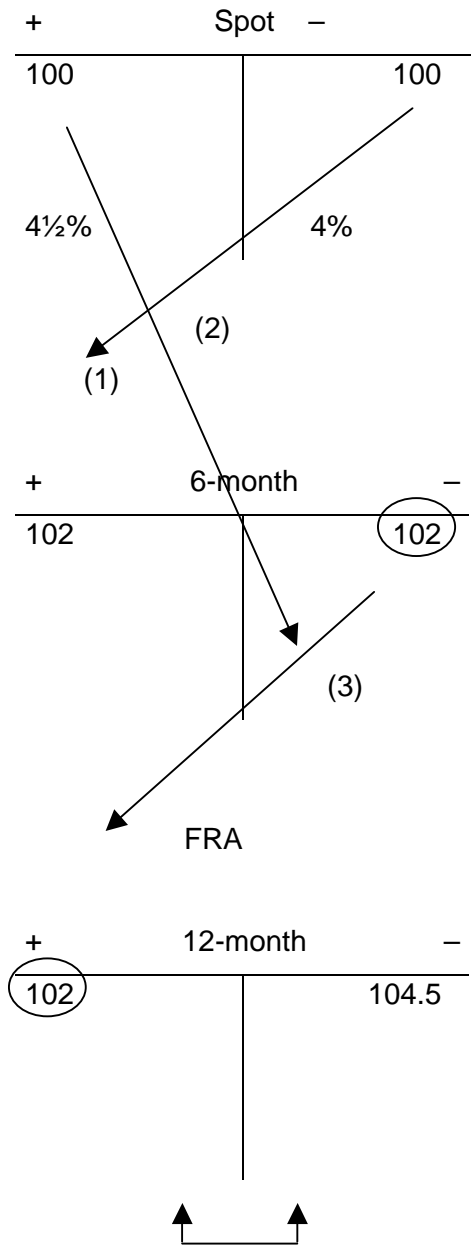
LIBOR at fixing date: 4.5 %

Days of FRA (interest period): 181

$$\text{CP} = \frac{(0.045 - 0.04) \cdot 100,000,000 \cdot \frac{181}{360}}{1 + \left(0.045 \cdot \frac{181}{360}\right)} = 245,827.05$$



The question is: Above which FRA rate is it possible to make risk-free profit?



$$\frac{2.5}{102} \cdot \frac{360}{180} = 4.90196\%$$

The FRA rate should be lower than 4.90 % side. If it is higher, arbitrage would be end 6 months, borrow 12 months and sell



According to the given rates of the deposits, the FRA rate should be between 4.53 % and 4.90 %.

Note:

- Since the deposit spreads lead to a relatively wide spread for the possible FRA rates, the FRAs are usually calculated on the basis of MM-Future rates (if such are available)
- In the example above, the demonstrated "arbitrage" still carries a risk since it is assumed that at the FRA's maturity date the deposit can be realized at the prevailing LIBOR rate.

### 2.4.3 The formulae to calculate FRA rates from deposit rates are

$$FRA_{bid} = \left\{ \frac{\left[ 1 + \left( \frac{r_{L(bid)} \cdot D_L}{B} \right) \right]}{\left[ 1 + \left( \frac{r_{S(offer)} \cdot D_S}{B} \right) \right]} - 1 \right\} \cdot \frac{B}{D_{FRA}}$$

$$FRA_{offer} = \left\{ \frac{\left[ 1 + \left( \frac{r_{L(offer)} \cdot D_L}{B} \right) \right]}{\left[ 1 + \left( \frac{r_{S(bid)} \cdot D_S}{B} \right) \right]} - 1 \right\} \cdot \frac{B}{D_{FRA}}$$

- $r_L$  = interest rate, long-term  
 $r_S$  = interest rate, short-term  
 $B$  = day basis of calculation  
 $D_L$  = number of days, long-term  
 $D_S$  = number of days, short-term  
 $D_{FRA}$  = number of days, FRA

**Example**

$$\text{FRA}_{\text{offer}} = \left[ \frac{1 + \left( \frac{0.045 \cdot 360}{360} \right)}{1 + \left( \frac{0.04 \cdot 180}{360} \right)} - 1 \right] \cdot \frac{360}{180} = 4.90196\%$$

$$\text{FRA}_{\text{bid}} = \left[ \frac{1 + \left( \frac{0.04375 \cdot 360}{360} \right)}{1 + \left( \frac{0.04125 \cdot 180}{360} \right)} - 1 \right] \cdot \frac{360}{180} = 4.53154\%$$

In a perfect market, there must be a link between the yield and the FRA rates. Therefore, the following principles can be derived.

- the steeper the yield curve, the higher the FRA rates
- the flatter the yield curve, the lower the FRA rates
- for a normal yield curve FRA rates are higher than the interest rate level
- for an inverse yield curve FRA rates are lower than the interest rate level

#### 2.4.4 Determination of FRA rates on the basis of interest rate futures

Using futures to compute FRA rates is common practice in the markets. To do this, the compound interest of the futures strip has to be taken into account. The FRA rates for non-IMM dates are computed by interpolation.

**Example**

Date: 8th of April 1997, USD futures

JUNE (M)	96.75 - 96.76
	June 18th - September 17th 1997 (91 days)
SEPTEMBER (U)	96.65 - 96.66
	September 17th - December 17th 1997 (91 days)
DECEMBER (Z)	96.50 - 96.51
	December 17th - March 18th 1998 (91 days)

#### Determination of 3-month term

One can derive the following:

FRA 2-5 IMM (June 18th - September 17th 1997) quoted on futures 3.24 - 3.25

FRA 5-8 IMM (September 17th - December 17th 1997) quoted on futures 3.34 - 3.35

FRA 8-11 IMM (December 17th - March 18th 1998) quoted on futures 3.49 - 3.50

FRA terms between these periods are interpolated.

*Note:* Please see the section about interest rate futures. To hedge futures vs. FRAs, it must be paid attention to the difference in the number of days: futures are always calculated with 90 days, while FRAs are also calculated with more days ( the FRA in the example with 91 days).

### Determination of 6-month term

With the help of futures contracts one can also determine the USD FRA 2·8 IMM rate:

$$\text{offer: } \left\{ \left[ \left( 1 + \left( 0.0325 \cdot \frac{91}{360} \right) \right) \cdot \left( 1 + \left( 0.0335 \cdot \frac{91}{360} \right) \right) \right] - 1 \right\} \cdot \frac{360}{182} = 3.31376\%$$

$$\text{bid: } \left\{ \left[ \left( 1 + \left( 0.0324 \cdot \frac{91}{360} \right) \right) \cdot \left( 1 + \left( 0.0334 \cdot \frac{91}{360} \right) \right) \right] - 1 \right\} \cdot \frac{360}{182} = 3.30368\%$$

Simplified:

FRA purchase of USD 50,000,000 at 3.31 for June 18th - December 17th 1997 can be hedged by buying 50 futures contracts in June at 96.76 and 50 contracts in September at 96.66.

### Determination of 9-month term

With the help of the 3-month FRA rates or the future rates the USD FRA 2 / 11 IMM rate can be determined:

$$\text{offer: } \left\{ \left[ \left( 1 + \left( 0.0325 \cdot \frac{91}{360} \right) \right) \cdot \left( 1 + \left( 0.0335 \cdot \frac{91}{360} \right) \right) \cdot \left( 1 + \left( 0.035 \cdot \frac{91}{360} \right) \right) \right] - 1 \right\} \cdot \frac{360}{273} = 3.39539\%$$

$$\text{bid: } \left\{ \left[ \left( 1 + \left( 0.0324 \cdot \frac{91}{360} \right) \right) \cdot \left( 1 + \left( 0.0334 \cdot \frac{91}{360} \right) \right) \cdot \left( 1 + \left( 0.0349 \cdot \frac{91}{360} \right) \right) \right] - 1 \right\} \cdot \frac{360}{273} = 3.38521\%$$